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An approach to time series modelling is described; it classifies the time series into one of three memory types (called no memory, short memory, and long memory), and then finds a whitening filter. When the time series is short memory one would like to identify the whitening filter type as AR, MA, or ARMA before parameter estimation. A new tool is introduced which can be used to diagnose both the memory type of a "time series," and the whitening filter type of a short memory time series. It is called prediction variance horizon function, and is defined by $PVH(h)=1-\sigma^2_{h,\infty}$, where $\sigma^2_{h,\infty}$ is the normalized mean square prediction error of infinite memory prediction h steps ahead.

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TIME SERIES MODEL IDENTIFICATION AND PREDICTION VARIANCE HORIZON

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An approach to time series modelling is described; it classifies the time series into one of three memory types (called no memory, short memory, and long memory), and then finds a whitening filter. When the time series is short memory one would like to identify the whitening filter type as AR, MA, or ARMA before parameter estimation. A new tool is introduced which can be used to diagnose both the memory type of a time series, and the whitening filter type of a short memory time series. It is called prediction variance horizon function, ~~and~~ is defined by $PVH(h) = 1 - \sigma^2_{h,\infty}$, where $\sigma^2_{h,\infty}$ is the normalized mean square prediction error of ~~infinite~~ memory prediction h steps ahead. To classify the model type of a time series, one uses the shape of PVH and the value of the horizon HOR (defined as the smallest value of h
less than or = for which $PVH(h) \leq 0.05$). The analysis of a real time series, called Freeze, is described.

0. TIME SERIES MODEL TYPES: NO MEMORY, SHORT MEMORY, AND LONG MEMORY

A stationary Gaussian time series $Y(t)$, with zero means, with covariance function $R(v) = E[Y(t)Y(t+v)]$ and correlation function $\rho(v) = R(v)/R(0)$ can be represented in general

$$Y(t) = S(t) + Z(t) \ ,$$

$$S(t) = \sum_j \{A_j \cos \frac{2\pi}{\tau_j} t + B_j \sin \frac{2\pi}{\tau_j} t\} \ ,$$

$$Z(t) = \sum_{k=0}^{\infty} \beta_k \varepsilon(t-k)$$

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for suitable periods τ_j, constants β_k, and uncorrelated random variables A_j, B_j, $\varepsilon(t)$. Thus $Y(t)$ is a sum of a scheme of hidden periodicities and an MA(∞) scheme. From a single realization of the time series one can hope to estimate β_k and σ^2, the variance of $\varepsilon(t)$; however one can only estimate the values of A_j and B_j in that realization, and not their distribution as random variables.

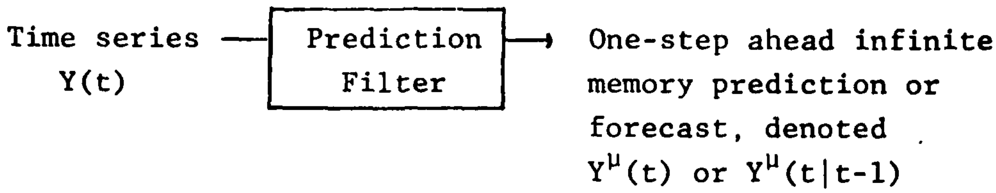
Both $S(t)$ and $Z(t)$ are stationary time series. When modeling $Y(t)$ by an ARMA (p,q) one is assuming that only the component $Z(t)$ is present. The ARIMA model introduced by Box and Jenkins (1976) can be regarded as an approach to modeling $Y(t)$ when seasonal or periodic components $S(t)$ are present (and also when trend terms are present). This paper proposes that while general models for seasonal and trend components cannot be defined, one can develop general diagnostics for their presence. Consequently, one is able to distinguish between stationary time series like $Z(t)$ which can be approximately modeled by an ARMA(p,q), and those whose models require terms like $S(t)$.

Parzen (1979), (1980) proposes that the basic strategy of time series model identification is to determine a *whitening filter* in such a way that time series decomposition filters can be obtained as interpretations of the whitening filter. A whitening filter is one which transforms the time series to white noise:

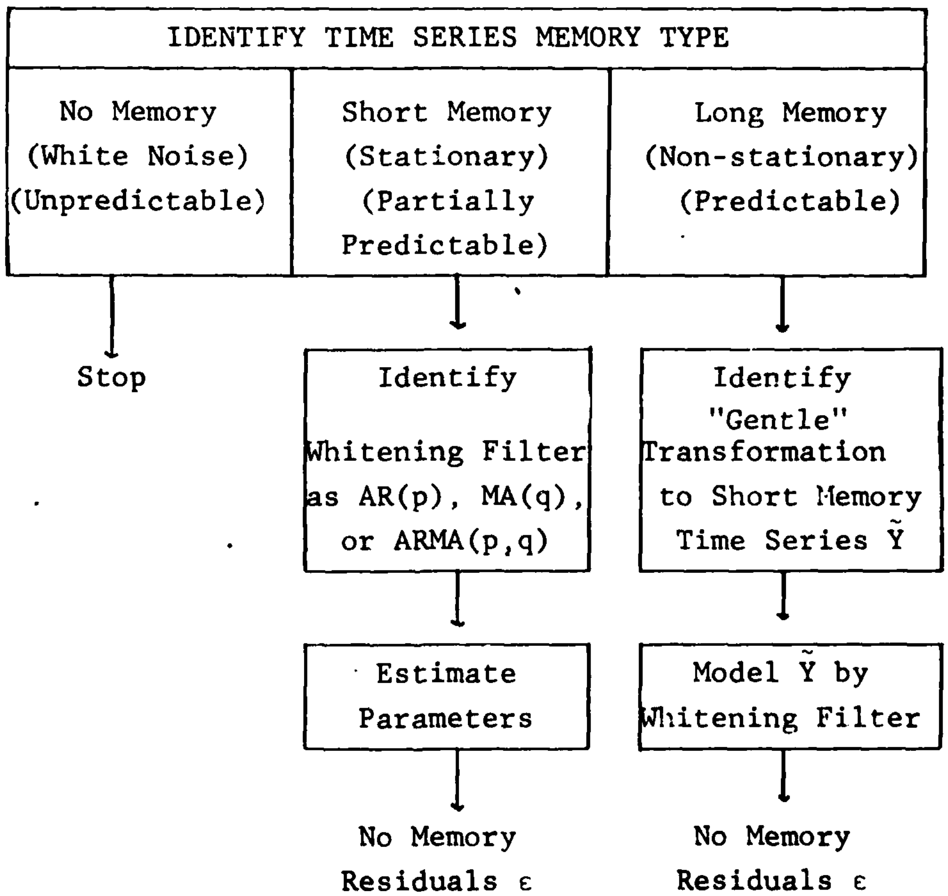
$$\begin{array}{c}\text{Time Series}\\ Y(t)\end{array} \longrightarrow \boxed{\begin{array}{c}\text{Whitening}\\ \text{Filter}\end{array}} \longrightarrow \begin{array}{c}\text{White noise, denoted}\\ \varepsilon(t) \text{ or } Y^{\nu}(t)\end{array}$$

It is closely related to a prediction filter which generates

a predictor, $Y^{\mu}(t)$ since $Y^{\nu}(t) = Y(t) - Y^{\mu}(t)$:



To identify the whitening filter of a time series, determine its *model memory* type, and identify an *iterative* model for the time series:



This paper describes various ways to define the three time series types, using (1) correlations, (2) spectral densities, (3) innovation variances, (4) spectral distribution

functions, (5) prediction variance horizon function, and (6) S-PLAY diagnostics. The various approaches do not lead to equivalent definitions; they are intended to illustrate the qualitative conclusions we seek to form about models obeyed by an empirical time series.

Let us illustrate how the concept of time series memory type changes our ways of describing time series models. Consider the model

$$g_2(L)Y(t) = h_2(L)\varepsilon(t)$$

where L is the lag operator defined by $L^k Y(t) = Y(t-k)$,

$$g_2(L) = I-1.97L + L^2 \ , \ h_2(L) = I-1.65L + .64L^2 \ .$$

A researcher might describe this as an ARMA(2,2) model, even though $g_2(L)$ has roots on the unit circle; $g_2(L)$ is approximately second differencing $I-2L + L^2$. If $g_2(L)$ were in fact second differencing, some researchers would describe the model as ARIMA(0,2,2). This paper proposes that more insight is obtained by writing the model as an iterated model

$g_2(L)Y(t) = \tilde{Y}(t)$,	Transform long memory Y to short memory $\tilde{Y}$,
$\tilde{Y}(t) = h_2(L)\varepsilon(t)$,	Transform short memory $\tilde{Y}$ to no memory ε.

The classification of a sample $\{Y(t) \ , \ t = 1, 2, \ldots, T\}$ will be based on various diagnostics, derived from basic sample statistics which constitute a generalized harmonic analysis; thus, the first step in the empirical analysis of a sample $\{Y(t), \ t = 1, 2, \ldots, T\}$ is the calculation of: (1) the sample spectral density function $\tilde{f}(\lambda)$, $-0.5 \le \lambda \le 0.5$

defined

$$\tilde{f}(\lambda) = \frac{\left| \sum_{t=1}^{T} Y(t) \exp(2\pi i\lambda t) \right|^2}{\sum_{t=1}^{T} Y^2(t)}$$

(2) the sample spectral distribution function $\tilde{F}(\lambda)$, $0 \leq \lambda \leq 0.5$, defined by

$$\tilde{F}(\lambda) = 2\int_0^{\lambda} \tilde{f}(\lambda')\, d\lambda' ,$$

(3) the sample correlation function $\hat{\rho}(v)$, $v = 0, 1, \ldots, T-1$, defined by

$$\hat{\rho}(v) = \frac{\sum_{t=1}^{T-v} Y(t)Y(t+v)}{\sum_{t=1}^{T} Y^2(t)}$$

$$= \int_{-0.5}^{0.5} e^{2\pi i\lambda v}\, \tilde{f}(\lambda) d\lambda .$$

Usually the foregoing quantities are computed for the mean-detrended time series $Y(t)-\bar{Y}$, $\bar{Y} = (1/T) \sum_{t=1}^{T} Y(t)$. When the time series has a strong trend component, a better detrending procedure is non-stationary subset autoregression which yields ARARMA models [Parzen (1981)]

1. MEMORY TYPE BY CORRELATION FUNCTIONS

The sample correlation function $\hat{\rho}(v)$ of a time series has the same mathematical properties as the correlation function $\rho(v)$ of a zero mean covariance stationary time series $Y(t)$.

In terms of $\rho(v)$, the definition of the three time series memory types is:

No Memory	Short Memory	Long Memory
$\sum_{v=1}^{\infty} \lvert\rho(v)\rvert = 0$	$0 < \sum_{v=1}^{\infty} \lvert\rho(v)\rvert < \infty$	$\sum_{v=1}^{\infty} \lvert\rho(v)\rvert = \infty$

Within short memory time series there are three types whose classification in terms of correlation functions is as follows:

MA(q) $\rho(v) = 0$ for $v > q$ (note MA(0) is white noise);

AR(p) There exist $\alpha_1, \ldots, \alpha_p$ such that

$$\rho(v) + \alpha_1\rho(v-1)+\ldots+\alpha_p\rho(v-p) = 0, \quad v > 0 ;$$

ARMA(p,q) There exist $\alpha_1, \ldots, \alpha_p$ such that

$$\rho(v) + \alpha_1\rho(v-1)+\ldots+\alpha_p\rho(v-p) = 0, \quad v > q .$$

Examples of AR and ARMA are: AR(1) $\rho(v) = \alpha^v$, $v > 0$; $\rho(v) - \alpha\rho(v-1) = 0$, $v > 0$: ARMA(1,1) $\rho(v) = \gamma\alpha^v$, $v > 0$; $\rho(v) - \alpha\rho(v-1) = 0$, $v > 1$, where $0 < \gamma < 1$.

Within long memory stationary time series some types are:

WHITE BANDLIMITED NOISE $\rho(v) = \dfrac{\sin 2\pi Bv}{2\pi Bv}$;

PERIODIC $\rho(v) = \cos\dfrac{2\pi}{p}v$;

PERIODIC PLUS WHITE NOISE $\rho(v) = \gamma\cos\dfrac{2\pi}{p}v$, $v > 0$.

The long memory time series $Y(t) = A \cos \frac{2\pi}{p}t + N(t)$ can

be transformed to a short memory time series by forming $\tilde{Y}(t) = (I-\phi L+L^2)Y(t)$ where $\phi = 2\cos(2\pi/p)$; we call this operation second-order quasi-differencing.

The definition in terms of correlations is intended only to introduce the three time series memory types. It is not our final definition, as correlations do not provide adequate means of identifying time series models.

2. MEMORY TYPE BY SPECTRAL LOG RANGE

The spectral density function $f(\lambda)$, $-0.5 \le \lambda \le 0.5$, of a stationary short memory time series is defined as the Fourier transform of $\rho(v)$:

$$f(\lambda) = \sum_{v=-\infty}^{\infty} e^{-2\pi i\lambda v}\,\rho(v)\ , \ -0.5 \le \lambda \le 0.5\ .$$

The spectral log range, and its memory types are defined by

$$\mathrm{SPLR} = \log \frac{\max f(\lambda)}{\min f(\lambda)}\ ,$$

No Memory	Short Memory	Long Memory
SPLR = 0	0 < SPLR < ∞	SPLR = ∞

To extend this definition to stationary time series whose correlation function $\rho(v)$ is not summable define, for any $T > 0$,

$$f_T(\lambda) = \frac{1}{T} \sum_{j,k=1}^{T} e^{-2\pi i\lambda j}\ e^{2\pi i\lambda k}\ \rho(j-k)$$

which is a non-negative function by the non-negative definite property of $\rho(v)$. Next

$$f_T(\lambda) = \sum_{|v|<T} e^{-2\pi i\lambda v} \left(1-\frac{|v|}{T}\right) \rho(v) ,$$

$$\left(1-\frac{|v|}{T}\right)\rho(v) = \int_{-0.5}^{0.5} e^{2\pi i\lambda v} f_T(\lambda)\, d\lambda .$$

We study the limits of these equations when $T \longrightarrow \infty$. When $\rho(v)$ is summable, $f_T(\lambda) \longrightarrow f(\lambda) \geq 0$. Otherwise, $\rho(v)$ is the limit of Fourier transforms of non-negative functions, and therefore there exists a spectral distribution function $F(\lambda)$, $-0.5 \leq \lambda \leq 0.5$, such that

$$\rho(v) = \int_{-0.5}^{0.5} e^{2\pi i\lambda v}\, dF(\lambda) .$$

It could be argued that in practice we are estimating not $f_T(\lambda)$ but

$$F_T(\lambda) = \int_{-0.5}^{\lambda} f_T(\lambda')\, d\lambda' \longrightarrow F(\lambda) .$$

The general definition of spectral log range is

$$\text{SPLR} = \lim_{T\to\infty} \log \frac{\max_\lambda f_T(\lambda)}{\min_\lambda f_T(\lambda)}$$

3. MEMORY TYPE BY PREDICTION VARIANCE

The time series memory type depends on the amount of variation in the spectral density $f(\lambda)$, which can be captured in a single criterion such as the following:

$$\sigma_\infty^2 = \exp - \int_{-0.5}^{0.5} \log f(\lambda) d\lambda \quad . \tag{1}$$

Note that $f(\lambda)$ is normalized so that $\int_{-0.5}^{0.5} f(\lambda)d\lambda = 1$. Then $0 \leq \sigma_\infty^2 \leq 1$ since by Jensen's inequality

$$\sigma_\infty^2 \leq \exp\{-\log \int_{-0.5}^{0.5} f(\lambda)d\lambda\} = e^0 = 1 \ .$$

If $f(\lambda) = 1$ identically (no memory or white noise time series) then $\sigma_\infty^2 = 1$. If $f(\lambda)$ approaches zero (long memory time series), then $\sigma_\infty^2 = 0$. Otherwise (short memory time series), $0 < \sigma_\infty^2 < 1$. It is shown in the prediction theory of stationary time series that σ_∞^2 defined by (1) equals the infinite memory one-step ahead mean square prediction error; we call σ_∞^2 the innovation variance. The classification of time series memory type in terms of σ_∞^2 is:

No Memory	Short Memory	Long Memory
$\sigma_\infty^2 = 1$	$0 < \sigma_\infty^2 < 1$	$\sigma_\infty^2 = 0$

To determine the value of σ_∞^2 , one uses the fact that

$$\sigma_\infty^2 = \lim_{m\to\infty} \sigma_m^2$$

where σ_m^2 is the finite memory m one-step ahead mean square prediction error. To compute σ_m^2 we solve for $\alpha_1, \ldots, \alpha_m$ the Yule-Walker equations

$$\rho(v) + \alpha_1\rho(v-1) + \ldots + \alpha_m\rho(v-m) = 0 \ , \quad v = 1, 2, \ldots, m;$$

then $\sigma_m^2 = 1 + \alpha_1\rho(1) + \ldots + \alpha_m\rho(m)$.

The coefficients $\alpha_1, \ldots, \alpha_m$ determine a prediction error time series

$$\tilde{Y}(t) = g_m(L)Y(t), \; g_m(z) = 1 + \alpha_1 z + \ldots + \alpha_m z^m \; ;$$

autoregressive spectral density estimator of order m

$$f_m(\lambda) = \sigma_m^2 \; |g_m(e^{2\pi i\lambda})|^{-2} \; , \; 0 \leq \lambda \leq 0.5 \; ;$$

and autoregressive spectral distribution function estimator of order m

$$F_m(\lambda) = 2\int_0^\lambda f_m(\lambda')d\lambda' \; , \quad 0 \leq \lambda \leq 0.5 \; .$$

From estimated correlations $\hat{\rho}(v)$, one solves $\hat{\rho}(v) + \hat{\alpha}_m(1)\hat{\rho}(v-1) + \ldots + \hat{\alpha}_m(m)\hat{\rho}(v-m) = 0$, $v = 1, \ldots, m$, $\hat{\sigma}_m^2 = 1 + \hat{\alpha}_m(1)\hat{\rho}(1) + \ldots + \hat{\alpha}_m(m)\hat{\rho}(m)$ for successive orders $m = 1, 2, \ldots$ (using fast algorithms); one forms transfer functions $\hat{g}_m(z) = 1 + \hat{\alpha}_m(1)z + \ldots + \hat{\alpha}_m(m)z^m$.

To choose an order $\hat{m}$ such that $\hat{\sigma}_{\hat{m}}^2 = \hat{\sigma}_\infty^2$ one computes order determining functions, such as AIC or CAT, whose absolute minimum and relative minimum are used to determine orders m of autoregressive estimators to be considered as "optimal".

Akaike information criterion is [see Akaike (1979)]

$$AIC(m) = \log \hat{\sigma}_m^2 + \frac{2m}{T}$$

An alternative version of criterion is given by Hannan and Quinn (1979).

Parzen CAT (Criterion Autoregressive Transfer Function) is a measure of the overall mean square relative error of g_m as an estimator of g_∞, the AR(∞) transfer function [see Parzen (1974), (1977)] . To define CAT let $\hat{\bar{\sigma}}_j^2 = \frac{T}{T-j} \hat{\sigma}_j^2$ which is called

an unbiased estimator of σ_j^2. Then

$$\mathrm{CAT}(m) = \frac{1}{T}\sum_{j=1}^{m} \hat{\sigma}_j^{-2} - \hat{\sigma}_m^{-2} .$$

For order 0, define AIC = 0, CAT = -1 . However if one wants to increase one's probability of correctly detecting white noise, one might adopt definitions such as CAT(0) $= -(1+\frac{1}{T})$, AIC(0) $= -\frac{1}{T}$.

In practice, identical conclusions are usually implied by these two criterion functions. The *best order* $\hat{m}$ is defined as the order at which the criterion function is minimized (if $\hat{m}=0$, the time series is considered to be white noise or no memory). The *second best order* $\hat{m}(2)$ is defined as the order at which the smallest relative minimum occurs which is not a global minimum. The approximating autoregressive scheme chosen by an order determining criterion, yields an estimate $\hat{\sigma}_{\hat{m}}^2$ of σ_∞^2 which provides a preliminary diagnostic of the memory type of a time series. An ad hoc rule is:

No Memory	Short Memory	Long Memory
$\hat{\sigma}_{\hat{m}}^2 > 1-\frac{4}{T}$	otherwise	$\hat{\sigma}_m^2 < \frac{8}{T}$

<u>Nonstationary autoregressive schemes</u>: A parallel approach to model identification is to fit autoregressive schemes which may be non-stationary. One minimizes

$$\sum_{t=m+1}^{T} \{Y(t) + \alpha(1)Y(t-1) + \ldots + \alpha(m)Y(t-m)\}^2$$

to form estimators $\hat{\alpha}(1), \ldots, \hat{\alpha}(m)$. Then form the squariance

$$S_m = \sum_{t=m+1}^{T} \{Y(t) + \hat{\alpha}(1)Y(t-1) + \ldots + \hat{\alpha}(m)Y(t-m)\}^2$$

and estimate σ^2(representing *normalized* mean square error) by

$$\hat{\sigma}_m^2 = \frac{1}{T-m} S / \frac{1}{T} \sum_{t=1}^{T} Y^2(t) .$$

If $\hat{\sigma}_m^2$ is near zero, which might be interpreted as indicating a model which fits the data well, one instead concludes that the time series is non-stationary and/or long memory, and one needs to model the residuals $\tilde{Y}(t) = Y(t) + \hat{\alpha}(1)Y(t-2) + \ldots + \hat{\alpha}(m)Y(t-m)$. If $\hat{\sigma}_m^2$ is near 1, one concludes that the time series is nearly white noise, and the coefficients $\hat{\alpha}(1), \ldots, \hat{\alpha}(m)$ are themselves undoubtedly not significantly different from zero.

4.. MEMORY TYPE BY SPECTRAL DISTRIBUTION FUNCTION

A quick diagnosis of the memory type of a time series can be obtained from the graph of the sample spectral distribution function $\tilde{F}(\lambda)$, $0 \leq \lambda \leq 0.5$, which together with $\hat{\rho}(v)$ is computed as the first step in our approach to time series analysis:

No Memory $\tilde{F}$ Uniform	Short Memory $\tilde{F}$ otherwise	Long Memory $\tilde{F}$ has sharp jumps

Spectral distribution functions play an important role in

judging the goodness of fit of a model. The mathematical fit of a model to data can be judged by the fit of a "smooth" function representing the model to a "wiggly" function representing the data. Spectral distribution functions are, in my opinion, ideal representing functions.

The orders selected by the minimum of an autoregressive criterion function should be regarded as hypotheses; one tests them by how well their autoregressive spectral distribution function fits the raw spectral distribution function, as measured by

$$\hat{F}(\lambda)-\tilde{F}(\lambda) = \int_0^{\lambda}\{\sigma_m^2|g_m(e^{2\pi iu})|^{-2}-\tilde{f}(u)\}du = \int_0^{\lambda}\{\hat{f}(u)-\tilde{f}(u)\}du.$$

In addition, the residuals of the autoregressive filter are tested for whiteness by examining

$$\int_0^{\lambda}\{\frac{1}{\sigma_m^2}|g_m(e^{2\pi iu})|^2\tilde{f}(u)-1\}du = \int_0^{\lambda} \frac{\tilde{f}(u)-\hat{f}(u)}{\hat{f}(u)}\, du \ .$$

5. MEMORY TYPE AND ARMA TYPE BY PREDICTION VARIANCE HORIZON

An extremely useful function for identification of a time series model type before parameter estimation is the *prediction variance horizon* PVH(h), h = 1, 2, ... It is defined in terms of the normalized mean square prediction error of infinite memory prediction h steps ahead:

$$\sigma^2_{h,\infty} = E[|Y^{\nu}(t+h|t)|^2] \div E[Y^2(t)], \quad Y^{\nu}(t+h|t) = Y(t)-Y^{\mu}(t+h|t),$$

$$Y^{\mu}(t+h|t) = E[Y(t+h)|Y(t), Y(t-1), \ldots] \ .$$

A formula for $\sigma^2_{h,\infty}$ is obtained by introducing the MA(∞) representation of Y(t): $Y(t) = \varepsilon(t) + \beta_1\varepsilon(t-1) + \ldots$. Then

$$\sigma^2_{h,\infty} = \sigma^2_\infty\{1 + \beta_1^2 + \ldots + \beta^2_{h-1}\} .$$

The graph of $\sigma^2_{h,\infty}$ increases monotonically from σ^2_∞ at h = 1 to 1 as h tends to ∞ . We define

$$PVH(h) = 1 - \sigma^2_{h,\infty} , \quad h = 1, 2, \ldots$$

and define horizon HOR to be the smallest values of h for which $PVH(h) \leq 0.05$ (whence $\sigma^2_{h,\infty} \geq .95$) .

The infinite moving average coefficients β_k are estimated by inverting the transfer function $g_m(z)$ of an approximating autoregressive scheme to obtain, for k = 1, 2, ...

$$\alpha_0\beta_k + \alpha_1\beta_{k-1} + \ldots + \alpha_k\beta_0 = 0 .$$

The value of the horizon HOR, and the shape of PVH, can be used to determine: (1) the memory type of a time series, and (2) if it is short memory, whether its model should be AR, MA, or ARMA. A long horizon indicates a long memory time series.

As an example, suppose one fits an AR(1): $Y(t)-\rho Y(t-1)=\varepsilon(t)$; when $\rho \doteq 1$ the time series is considered long memory since $\sigma^2_\infty = 1 - \rho^2 \doteq 0$, $PVH(h) \doteq \rho^{2h}$, $HOR \doteq (\log .05)/2 \log \rho \doteq \infty$. When $\rho \doteq 0$, the time series is considered no memory. When $0 < \rho < 1$, the time series is considered short memory.

The classification of memory type by prediction horizon HOR is:

No Memory	Short Memory	Long Memory
$HOR \doteq 0$	$0 < HOR < \infty$	$HOR \doteq \infty$

By HOR $\doteq \infty$, we mean HOR is comparatively large: experiments lead us to conclude that one should compare HOR with the order ORD of the approximating autoregressive scheme. Let HOR/ORD denote the ratio of HOR to ORD; identify time series as follows: If HOR/ORD ≤ 1, then MA(q), with $q \leq$ HOR-1. If HOR/ORD ≥ 4(say); and PVH decays slowly, then long memory. If PVH declines smoothly and exponentially, then an AR(p) is indicated. If PVH has "bends", then ARMA. If PVH has many level stretches with period τ, then an ARMA model is indicated of the form

$$Y(t) = \frac{I+\beta_1 L+\beta_2 L^2+\ldots+\beta_q L^q}{I-\alpha_\tau L^\tau}\,\varepsilon(t) .$$

The final identification of the orders p and q should be by parameter estimation or by use of S-arrays.

6. MEMORY TYPE AND ARMA TYPE BY S-ARRAYS

Gray, Kelley, and McIntire (1978) define a double sequence *S-Array*: $S_m^*(-q)$, ..., $S_m^*(-1)$, $S_m^*(0)$, $S_m^*(1)$, ..., $S_m^*(q)$, $m = 1, 2, \ldots, p$, whose constancy patterns are in 1-1 correspondence with ARMA schemes, and more generally with characteristics of sequences satisfying

$$\rho(v) + \alpha_1\rho(v-1) + \ldots + \alpha_p\rho(v-p) = 0 , \quad v > q .$$

The generalized partial correlation is defined by $\pi_m^{(n)} = -a_m(m)$, where $a_m(m)$ is the value of the last autoregressive coefficient in an ARMA(m,n) scheme obtained by solving *higher order* Yule

Walker equations

$$\rho(j) + a_m(1)\rho(j-1)+\ldots+ a_m(m)\rho(j-m) = 0, \; j = m + 1, \ldots \; n+m.$$

Woodward and Gray (1980) show that

$$|\pi_m(n)| = |S_m^*(n)/S_m^*(-(n+1))| \;, \quad n = 0, 1, \ldots .$$

The display which we call S-PLAY combines various statistical diagnostics with a new way of displaying S^*-arrays. For frequency λ (equal to 0 or 0.5) S-PLAY displays for each autoregressive order p = 1, 2, ...

- V — Innovation variance $\hat{\sigma}_p^2$
- PL — Partial correlation $|\pi_p^{(0)}|$
- -AIC — The value of -AIC(p)
- SPEC — Autoregressive spectral estimator $\hat{f}_p(\lambda)$ at frequency λ.
- GSP1, GSP2 — G-spectral estimators at frequency λ
- P2D — Minimum value of 2nd differences of generalized partial correlations $\pi_p^{(n)} - 2\pi_p^{(n+1)} + \pi_p^{(n+2)}$; value of n at which minimum attained is printed as value of symbol M. If one adopts p as autoregressive order, one might consider M as moving average order.

It should be noted that the rows of S-PLAY are the columns of the shifted S-Array as defined by Woodward and Gray (1980).

The constancy patterns in the S-array which are characteristic of an ARMA(p,q) are described below, as are the patterns characteristic of long memory. S-PLAY provides diagnostics which can be used to confirm our conclusions about a time series model type as follows:

No Memory	Short Memory	Long Memory
Infinities in Column - 1	ARMA(p,q) if row p, column q, column -(q+1) have constant values as described in (a) and (b)	Constant row 1 (trend) Constant row 2 (seasonal)

(a) $C_2 = S_p(-(q+1)) = S_p(-(q+2)) = \ldots$ and $C_1 = S_p(q) = S_p(q+1) = \ldots$ implies that $\pi_p^{(q)} = \pi_p^{(q+1)} = \ldots$; in words, if one solves for the coefficients of a p-th order autoregressive scheme using high lag Yule-Walker equations of lag q, q + 1, ..., one obtains the same value for the p-th coefficient $a_p(p)$ from all of these sets of equations.

(b) $-C_1 = S_q(p+1)$, $C_1 = S_q(p+2)$, ..., $(-1)^i C_1 = S_q(p+i)$, $\pm\infty = S_{p+i}(-(q+1))$ for $i \geq 1$, implies that $\pi_{p+1}^{(q)} = \pi_{p+2}^{(q)} = \ldots = 0$; in words, if one solves (for i=1, 2, ...) for the coefficients of a (p+i)-th order autoregressive scheme using high lag Yule-Walker equations of lag q, one obtains value 0 for the last coefficient $a_{p+i}(p+i)$.

It should be noted that experience and judgement is required to decide when a pattern of approximate constancy exists in an S-array of an observed time series. Further the failure of such patterns to exist should be expected as most time series are not exactly an ARMA process, but at best are approximately modelled by an ARMA process.

7. CASE STUDY OF AN EMPIRICAL TIME SERIES ANALYSIS

The time series Y, called *Freeze*, and graphed in Fig. 27, represents minimum temperatures (in degrees centrigrade) over 10 day intervals (with some 11 day intervals). There are 36 values per year. The sample size T = 992.

Step 1. <u>Autoregressive Analysis of Y</u>

Compute sample mean $\bar{Y} = 5.02$, sample variance $\hat{R}(0) = 27.4$. For $Y(t)-\bar{Y}$, compute sample spectral density $\tilde{f}(\lambda)$, sample correlations $\hat{\rho}(v)$ and the sample spectral distribution function $\tilde{F}(\lambda)$. $\hat{\rho}$(Fig. 2) has a strictly periodic appearance, with periodicity 36. Some noteworthy maximum values are $\hat{\rho}(1) = .7163$, $\hat{\rho}(36) = .6765$, $\hat{\rho}(.73) = .6314$. $\tilde{F}$(Fig. 1) rises linearly except for a sharp jump of .66 at a frequency corresponding to a period of about 36. The character of $\hat{\rho}$ and $\tilde{F}$ leads us to suspect a model of periodic signal S(t) plus white noise N(t):

$$Y(t) = S(t) + N(t) , \tag{1}$$

$$S(t) = A \cos \frac{2\pi}{p}t + B \sin \frac{2\pi}{p}t , \quad p = 36 . \tag{2}$$

A model of quasi-periodic signal S(t) would be

$$S(t) - \phi S(t-1) + S(t-2) = \delta(t) \text{ white noise} \tag{3}$$

where $\phi = 2 \cos 2\pi/p = 1.97$ for $p = 36$. The variance of $N(\cdot)$ is approximately 30% of the variance of $Y(\cdot)$.

Next one solves Yule-Walker equations in $\hat{\rho}(v)$ for successive orders m = 1, 2, ..., M, where M is chosen in the range $T/10 \le M \le 3T/4$, depending on the size of T. Here M = 80. Graphs of $\log \hat{\sigma}_m^2$, AIC, and CAT are displayed in Figs. 3, 4. CAT chooses $\hat{m} = 36$, $\hat{m}(2) = 39$ as best and second best orders. It

should be noted that for Freeze AIC and CAT are unusually flat in the vicinity of their minimum order. The prediction variance horizon (Fig. 5) indicates Freeze is a long memory time series; the order 1 rows of S-PLAY indicate long memory; the innovation variance $\hat{\sigma}^2_{36} = .32$ does not indicate long memory. The autoregressive spectral density estimator $\hat{f}(\lambda)$ of order 36 in Fig. 6 and second best order 39 in Fig. 8 indicates a sharp peak at frequency 1/36 (with period 36); Fig. 7 shows that $\hat{F}$ matches $\tilde{F}$.

Step 2. Analysis of Autoregressive Residuals

The residuals $\tilde{Y}(t) = g_{36}(L)Y(t)$ of the AR(36) scheme yield the conclusion that they are white noise under autoregressive analysis (Figs. 9-16).

Step 3. Transformation of Long Memory Y to Short Memory $\tilde{Y}$

When Y has a long horizon, we seek to find a suitable transformation, to short memory; we choose

$$\tilde{Y}(t) = g_2(L)Y(t) \ , \quad g_2(L) = I - 1.97L + L^2 \ .$$

Its variance 42.7 is larger than that of Y(t), which one can explain by the approximate calculation Var $[Y(t) - 2Y(t-1) + Y(t-2)] = 6R(0) = 8R(1) + 2R(2)$. The raw spectral distribution function (Fig. 18) of $\tilde{Y}(\cdot)$ suggests high frequency content. Its correlation function (Fig. 17) suggests a MA(2) model since $\hat{\rho}(1) = -.6405$, $\hat{\rho}(2) = .1211$ and these are the only correlations much greater than $.06 \doteq 2/\sqrt{T}$.

The AIC and CAT determined (Fig. 19, 20) optimal autoregressive approximating order for $\tilde{Y}$ is $\hat{m} = 15$; note that AIC is much flatter than is CAT near the minimum $\hat{\sigma}^2_{15} = .24$.

PVH (Fig. 21) decreases very quickly to zero, and HOR = 3 suggests MA(2) . S-PLAY does not seem to yield definite conclusions, as it should not when the true model is a pure moving average.

The model $\hat{g}_{15}(L)\tilde{Y}(t) = \varepsilon(t)$ yields one-step ahead predictions with average squared error 8.7 whose ratio to the variance of $Y(t)$ is .32, exactly equal to the prediction variance of the AR(36) scheme fitted to $Y(t)$. One can conclude that the iterative analysis

$$Y \rightarrow \tilde{Y} = g_2 Y \rightarrow \varepsilon = g_{15}\tilde{Y} = g_{15} g_2 Y$$

has resulted in an AR(17) filter with the same forecasting (and the same spectral estimation) properties as the AR(36) model $g_{36}(L)Y = \varepsilon$. As a model the iterated AR(2), AR(15) model has more insight than the AR(36) model.

Step 4. <u>MA Analysis of $\tilde{Y}$</u>

The model $Y(t) = \varepsilon(t) + \beta_1\varepsilon(t-1) + \beta_2\varepsilon(t-2)$, an MA(2), suggested by the prediction variance horizon function and correlations of $\tilde{Y}(\cdot)$, needs to be confirmed and its parameters estimated. An alternative approach to identifying the ARMA type is to examine the successive models for $\tilde{Y}$ determined by our subset ARMA algorithm. The first model is MA(1), the second model is MA(2). Thereafter various ARMA models are determined whose residual variances do not decrease much. Consequently we fit $\tilde{Y}(t)$ by

$$\tilde{Y}(t) = \varepsilon(t) - 1.6475\ \varepsilon(t-1) + .6382\ \varepsilon(t-2)$$

with residual variance .26 (compared with .24 for AR(15)).

One concludes that an overall model for Y is an ARMA(2,2) model; estimators of its parameters are

$$Y(t)-1.97Y(t-1)+Y(t-2) = \varepsilon(t)-1.6475\varepsilon(t-1)+.6382\varepsilon(t-2) \ .$$

The spectral density of the MA(2) model for $\tilde{Y}$ is in Fig. 23. The spectral density of the ARMA model for Y is in Fig. 24.

Step 5. Signal Plus Noise Interpretation of ARMA Model

While the ARMA(2,2) model may not be most satisfactory for forecasting, or for spectral estimation, it is most satisfactory for insight. It suggests that the original time series Y(t) satisfies (1) and (3), since $\delta(t) + N(t) - 1.97\, N(t-1) + N(t+2)$ defines an MA(2) .

A question which the Freeze time series may illustrate is the question of how well autoregressive spectral estimators do when applied to data which is a moving average. Fig. 25 displays window estimators of the spectral density of Y(t), using a Parzen window with truncation points 16, 32, 64; the best of these truncation points is 32, and it agrees with the AR(15) spectral estimator in Fig. 22. The periodogram being smoothed is shown in Fig. 26.

Summary

The models we have fitted to Y(t) are as follows:

Steps 1,2:	AR(36)	$g_{36}(L)\{Y(t)-\bar{Y}\} = \varepsilon(t)$,
Step 3:	AR(2),AR(15)	$g_2(L)\{Y(t)-\bar{Y}\} = \tilde{Y}(t)$,
		$g_{15}(L)\tilde{Y}(t) = \varepsilon(t)$
Step 4:	AR(2),MA(2)	$g_2(L)\{Y(t)-\bar{Y}\} = \tilde{Y}(t)$,
		$\tilde{Y}(t) = h_2(L)\varepsilon(t)$

Step 5: $Y(t) = \bar{Y} + S(t) + N(t)$, $S(t) - 1.97S(t-1) + S(t-2) = \delta(t)$.

COEFFICIENTS α_j OF AR(36) MODEL $\Sigma\alpha_j Y(t-j) = \varepsilon(t)$

FOR FREEZE SERIES $Y(t)$, $t = 1, \ldots, 992$

1-5	-0.1897	-0.0789	-0.0831	-0.0774	0.0082
6-10	-0.0457	-0.0052	-0.0385	0.0185	0.0522
11-15	0.0095	-0.0103	0.0337	0.0888	0.0373
16-20	0.0311	0.0400	-0.0047	0.0358	0.0230
21-25	-0.0196	0.0534	0.0493	0.0341	-0.0178
26-30	-0.0138	0.0234	0.0077	-0.0371	-0.0117
31-35	-0.0708	-0.0319	0.0131	-0.0722	-0.0410
36	-0.0456		Residual Variance = 0.32125		

COEFFICIENTS α_j AR(15) MODEL $\Sigma\alpha_j \tilde{Y}(t-j) = \varepsilon(t)$

FOR TRANSFORMED FREEZE SERIES $\tilde{Y}(t) = Y(t) - 1.97Y(t-1) + Y(t-2)$

1-5	1.6390	2.0567	2.2628	2.2869	2.2303
6-10	2.0593	1.8400	1.5492	1.2572	1.0049
11-15	0.7548	0.4939	0.2857	0.1349	0.0482
			Residual Variance = 0.24223		

Successive Subset ARMA Models for $Y(t)$ were:

$$\tilde{Y}(t) = \varepsilon(t) - 1.647\ \varepsilon(t-1)$$

$$\tilde{Y}(t) = \varepsilon(t) - 1.647\ \varepsilon(t-1) + .638\ \varepsilon(t-2)$$

$$\tilde{Y}(t) = \varepsilon(t) - 1.647\ \varepsilon(t-2) + .638\ \varepsilon(t-2) - .059\ \tilde{Y}(t-5),$$

Residual variances are .359, .262, .259.

8. CONCLUSIONS

The final model fitted to a time series will often be an iterated model (with symbolic transfer functions G and g_∞)

$$Y(t) \longrightarrow \boxed{G} \longrightarrow \tilde{Y}(t) \longrightarrow \boxed{g_\infty} \longrightarrow \varepsilon(t) \text{ white noise}$$

where $\tilde{Y}(t)$ is the results of a transformation chosen to transform a long memory to a short memory one. One should always analyze the residuals of an approximating autoregressive filter to determine if they are white noise.

Autoregressive analysis by Yule-Walker equations yields a stationary autoregressive scheme; a non-stationary autoregressive scheme may be fit by estimating its coefficients by ordinary least squares. Parzen (1981) introduces the terminology ARARMA scheme for the iterated time series model with G non-stationary autoregressive, and g_∞ ARMA; an ARIMA scheme corresponds to a pure differencing operator for G.

A model frequently fitted to monthly economic time series is the so-called "airline" model [see Parzen (1979)]:

$$(I-L)(I-L^{12}) \cdot Y(t) = (I-\Theta_1 L)(I-\Theta_{12} L^{12}) \; \varepsilon(t) \; .$$

It seems doubtful that this model would be judged adequate by the criteria proposed in this paper.

It may happen that long memory components continue to be present even after several iterations; thus the final model might be of the form

$$Y(t) \longrightarrow \boxed{} \longrightarrow \tilde{Y}^{(1)}(t) \longrightarrow \boxed{} \longrightarrow \tilde{Y}^{(2)}(t) \longrightarrow \boxed{} \longrightarrow \varepsilon(t)$$

The iterated filter model can be used for forecasting, for spectral analysis, and for model interpretation.

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S-PLAY T = 992 FREEZE -10

P V	PZD PL	-AIC M	GSP1 SPEC	 GSP2	 0	-1 	 1	-2 	 2	-3 	 3	-4 	 4	-5 	 5
1	.04	0.72	0.25			-2.40		-2.09		-2.00		-2.13		-2.24	
.467	.72	2	0.17	0.25	-1.72		-1.52		-1.55		-1.69		-1.31		-1.50
2	***	0.81	0.25			-4.03		62.00		1.00		3.20		23.2	
.444	.30	0	0.31	0.25	1.20		1.93		-7.49		5.39		1.99		1.17
3	.03	0.83	0.25			-5.50		-14.09		-35.00		-4.10		-1.1	
.434	.15	0	0.23	0.22	-1.33		-1.54		-5.65		-9.05		1.07		-3.65
4	.03	0.53	0.27			-69.63		7.00		-21.04		7.42		2.25	
.434	.02	4	0.24	0.30	1.35		-0.61		3.42		-2.33		1.29		0.50
5	.03	0.84	0.30			16.72		13.75		9.03		2.70		1.00	
.428	.11	0	0.30	0.30	-1.20		-2.44		-1.51		-0.35		-0.10		-0.31
6	.03	0.85	0.20			12.03		-46.19		-7.53		3.01		-3.50	
.424	.10	2	0.24	0.29	1.33		3.13		-1.30		0.06		0.73		-0.51
7	.05	0.87	0.25			7.12		30.32		10.04		8.90		0.00	
.413	.10	0	0.33	0.29	-1.12		-2.20		-0.34		-0.34		-0.10		-0.02
8	.00	0.89	0.29			5.04		-34.58		-64.72		-6.51		-9.42	
.406	.13	4	0.25	0.23	1.27		2.77		0.64		-3.02		3.31		3.21
9	.01	0.92	0.29			5.70		47.42		39.28		37.97		36.40	
.393	.10	2	0.30	0.29	-1.04		-2.31		-4.03		-4.27		-4.20		-4.37
10	.07	0.95	0.29			8.55		32.02		21.13		432.40		-20.14	
.379	.19	1	0.25	0.29	1.23		1.90		1.40		3.94		-4.39		-10.29
11	.04	0.97	0.29			7.70		-113.27		-39.01		-229.01		-698.43	
.372	.14	0	0.33	0.29	-1.00		-2.03		-0.40		-4.34		15.02		40.17
12	.05	0.93	0.29			10.27		-47.53		-38.39		61.15		-6.10	
.367	.12	3	0.20	0.29	1.19		2.49		2.13		0.51		-0.37		-4.43
13	.33	1.00	0.29			7.00		393.00		50.03		-75.71		-90.03	
.360	.14	3	0.35	0.28	-1.02		-2.43		-34.02		-5.07		4.10		5.50
14	.01	1.03	0.29			0.02		21.33		14.41		22.44		31.32	
.343	.17	3	0.24	0.29	1.19		1.02		1.20		1.74		2.92		3.81
15	.02	1.04	0.27			9.44		-53.10		-34.25		-2.49		0.50	
.344	.11	3	0.31	0.29	-1.00		-2.09		-2.70		0.39		-0.70		-2.40
16	.03	1.04	0.29			11.02		-210.20		40.27		20.11		5.74	
.341	.10	2	0.25	0.29	1.10		2.19		-3.50		0.70		1.10		2.15
17	.03	1.05	0.29			10.90		31.17		-22.40		-65.56		-47.25	
.336	.10	0	0.30	0.29	-1.05		-1.57		0.03		-1.03		-2.49		-2.05
18	.03	1.05	0.30			24.43		-23.94		-14.29		-102.35		-70.13	
.337	.04	1	0.20	0.29	1.10		2.94		0.75		1.56		1.10		-3.06
19	.01	1.00	0.40			13.01		40.55		32.05		593.53		92.70	
.335	.03	2	0.35	0.29	-1.02		-1.31		-2.00		-1.02		0.92		-1.01
20	.00	1.00	0.20			10.03		30.38		-21.90		-44.94		-159.70	
.334	.00	3	0.29	0.28	1.07		1.31		-3.10		2.44		4.83		10.23
21	.11	1.06	0.20			64.33		-24.41		-3.09		-50.27		40.20	
.334	.02	1	0.30	0.20	-1.06		-0.59		-0.37		-1.70		4.50		-0.31
22	.00	1.00	0.20			13.77		30.53		29.05		13.05		12.55	
.332	.07	4	0.20	0.28	1.13		1.03		2.02		2.23		1.37		1.45
23	.27	1.00	0.20			21.95		131.02		-37.43		-3.20		-81.44	
.331	.03	0	0.29	0.25	-1.00		-1.00		-10.07		-0.02		-1.00		7.82
24	.00	1.00	0.20			44.13		-2.09		33.32		52.01		15.97	
.331	.02	3	0.27	0.25	1.10		-0.70		2.03		1.05		1.03		3.80

Fig. 1

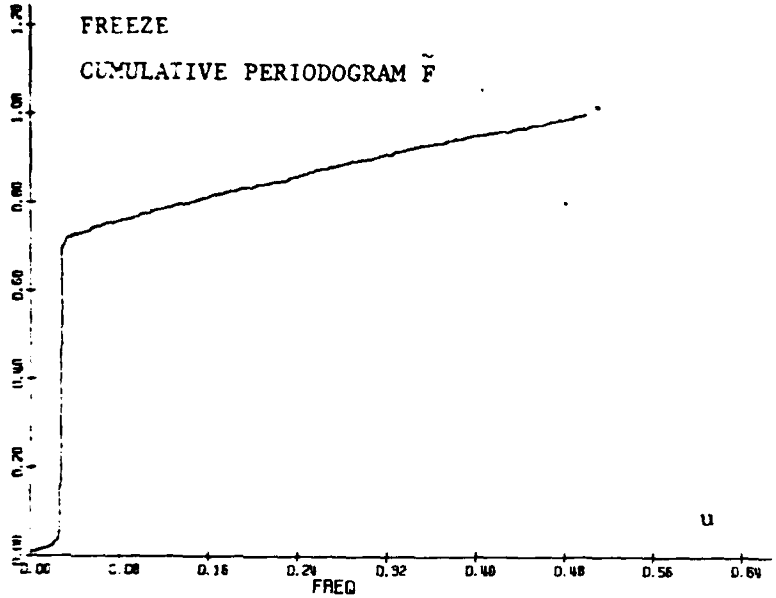


Fig. 2

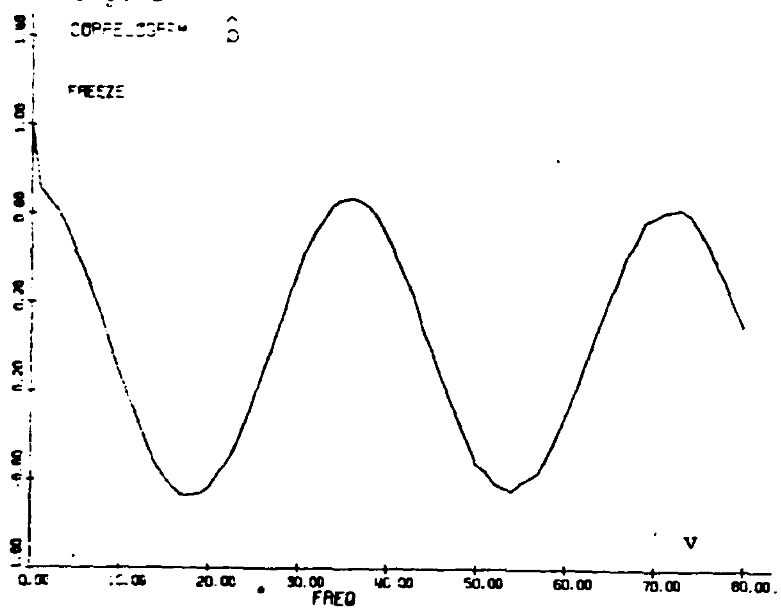


Fig. 3

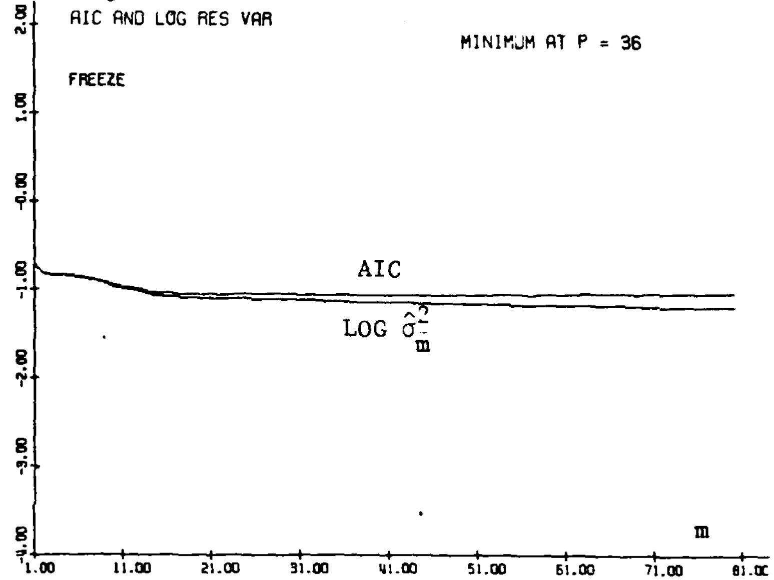
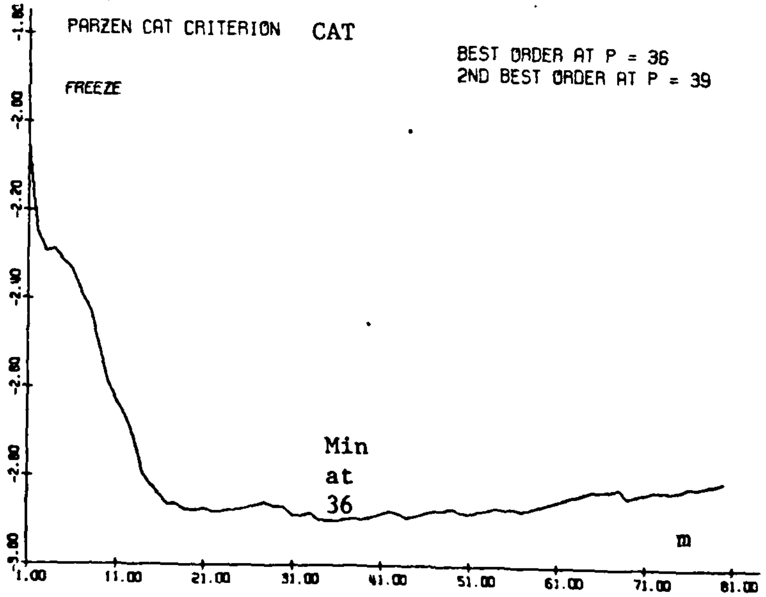


Fig. 4



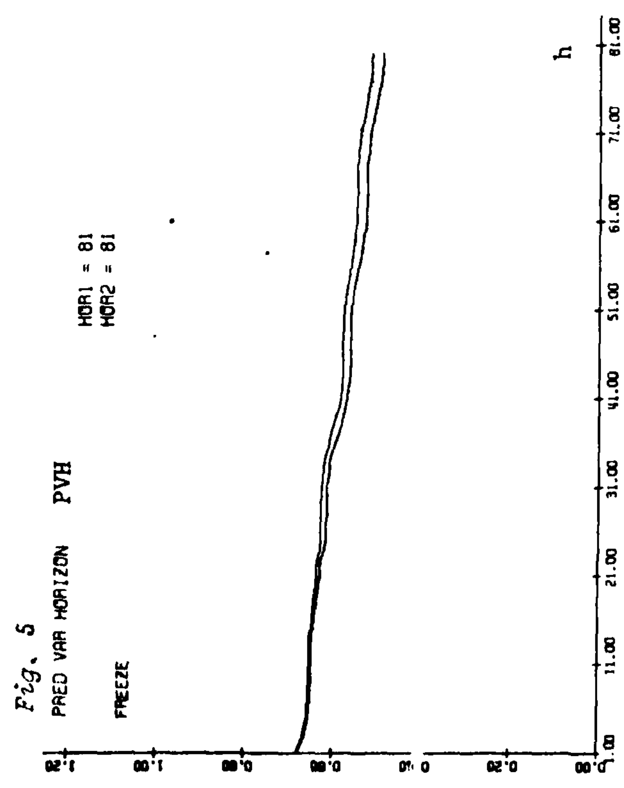


Fig. 5

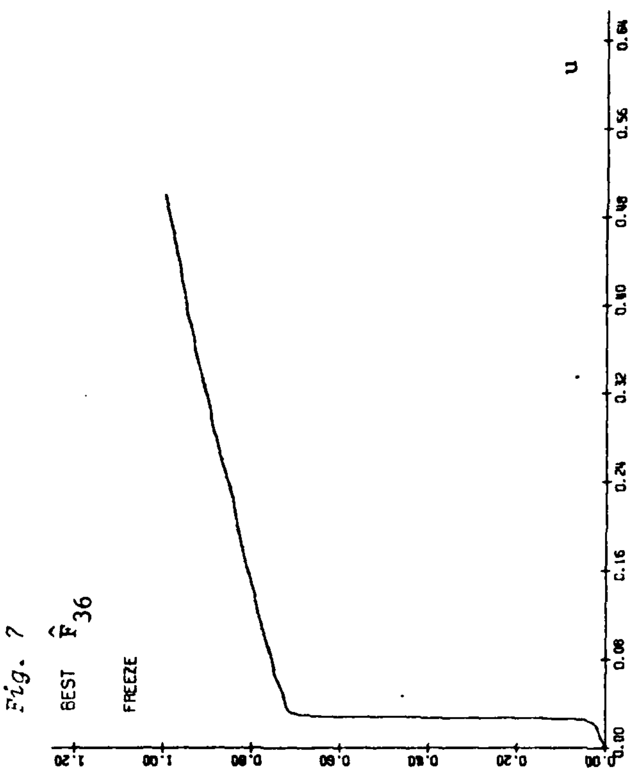


Fig. 7

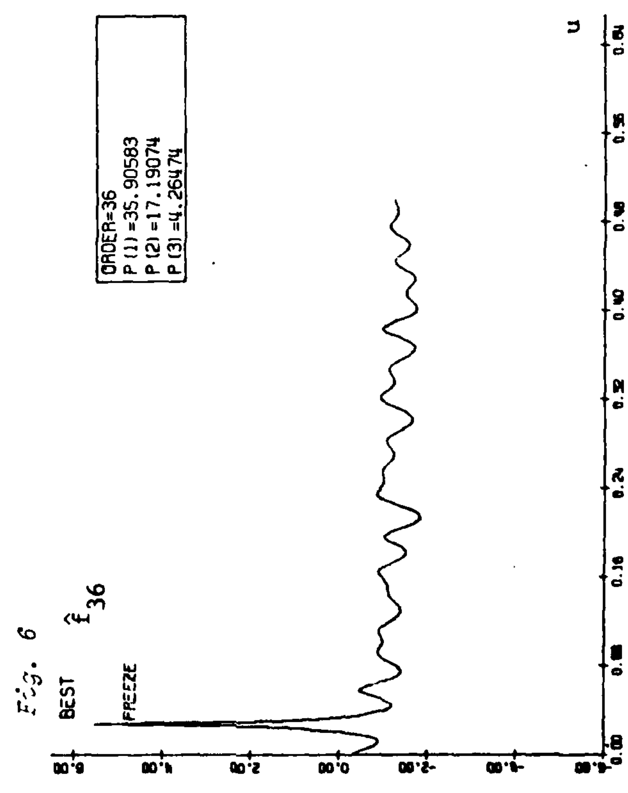


Fig. 6

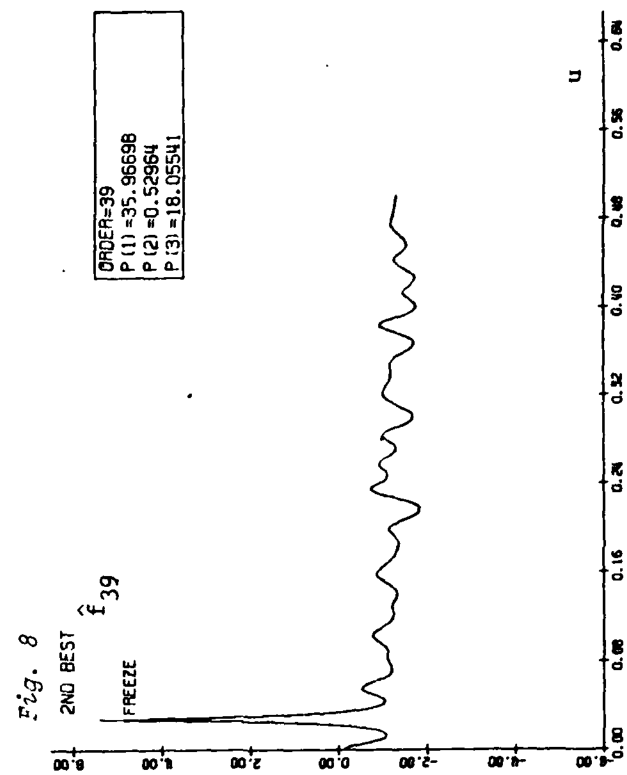


Fig. 8

Fig. 9

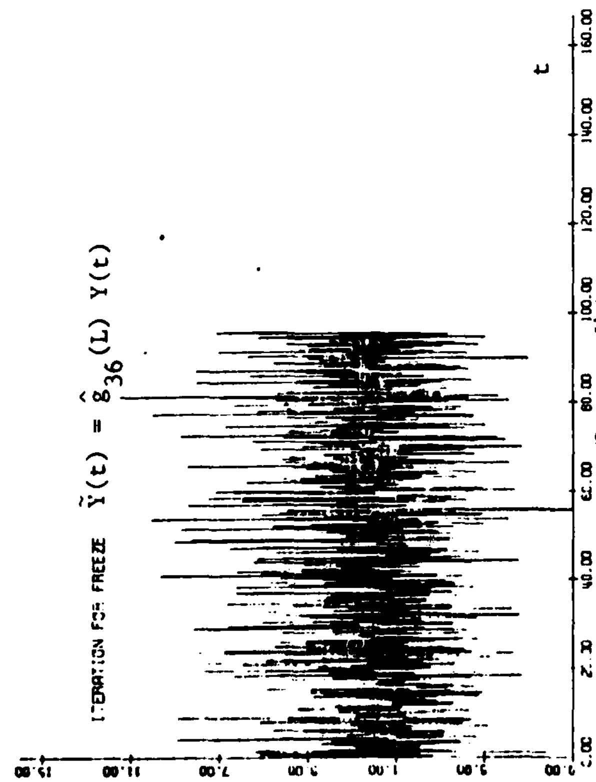


Fig. 10

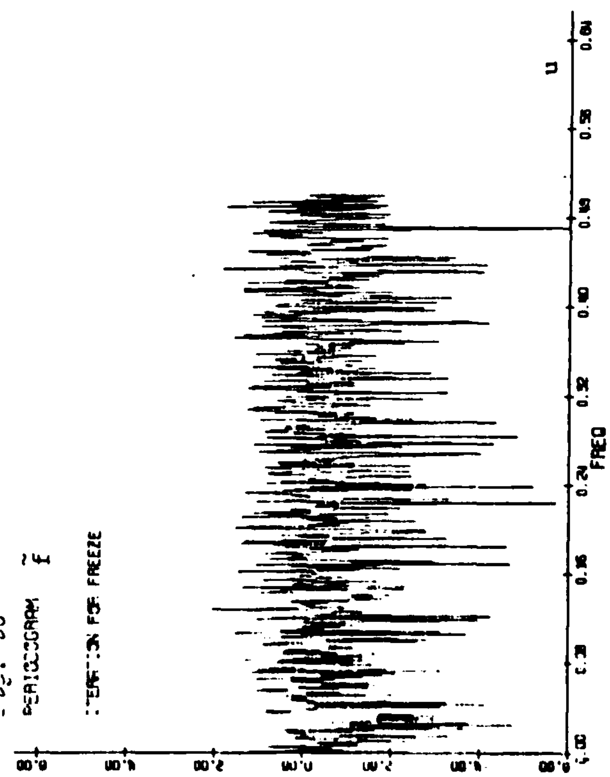


Fig. 11

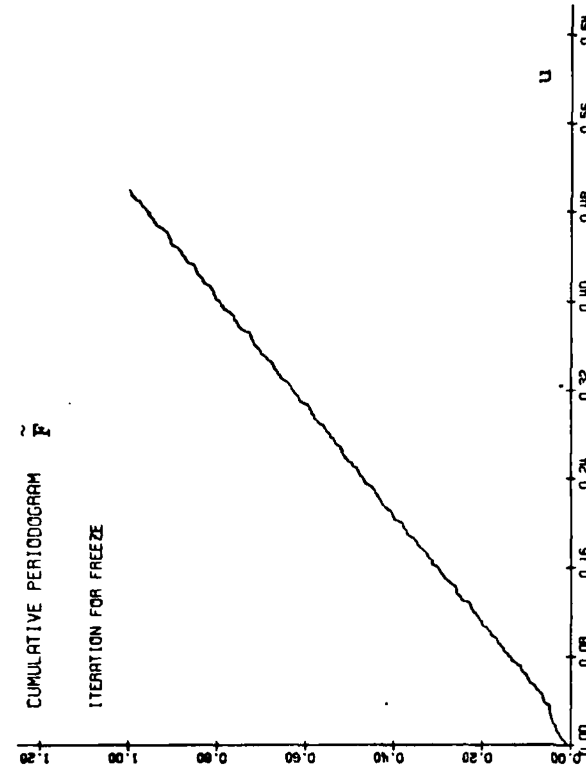


Fig. 12

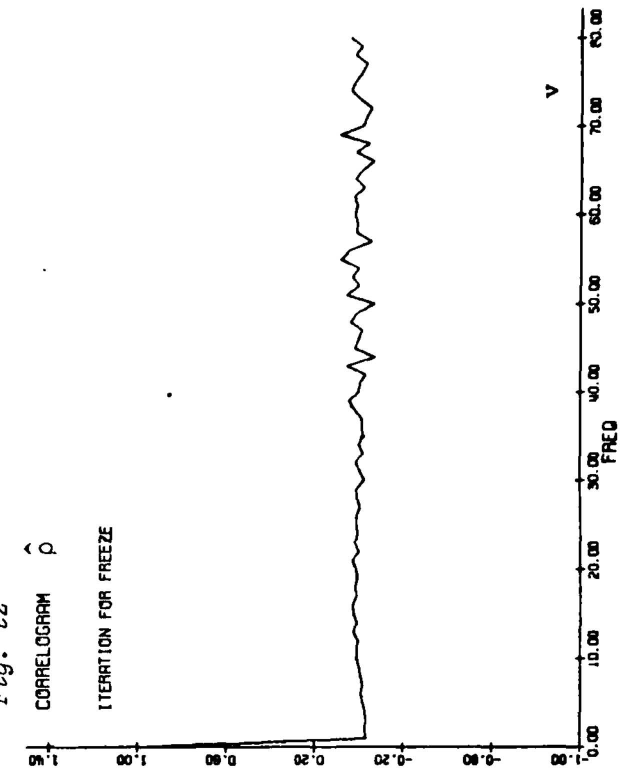


Fig. 13

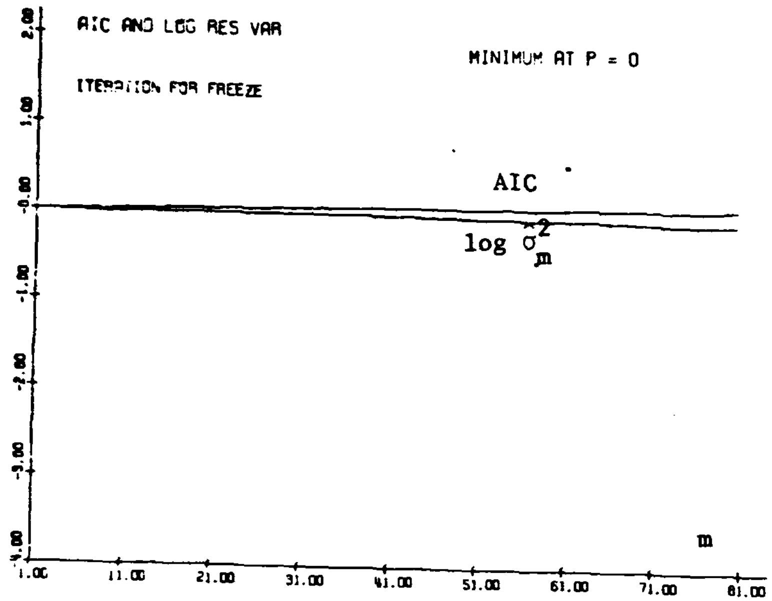


Fig. 14

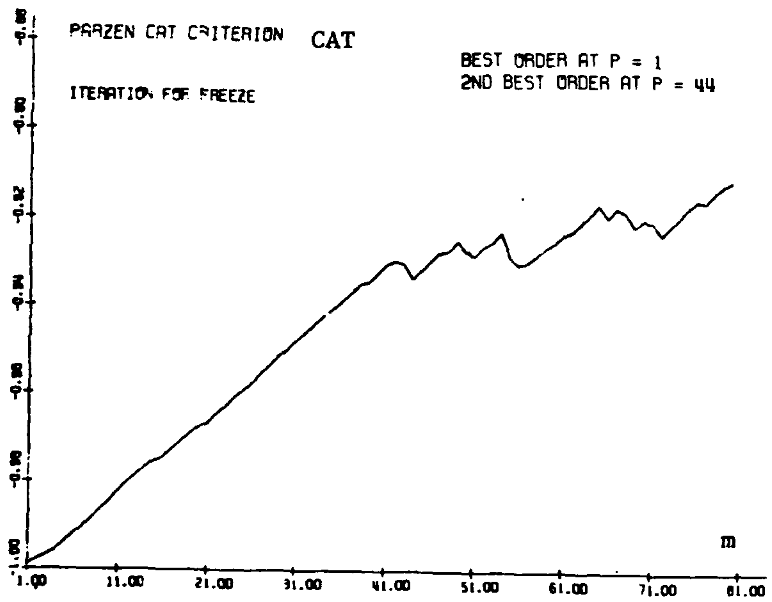


Fig. 15

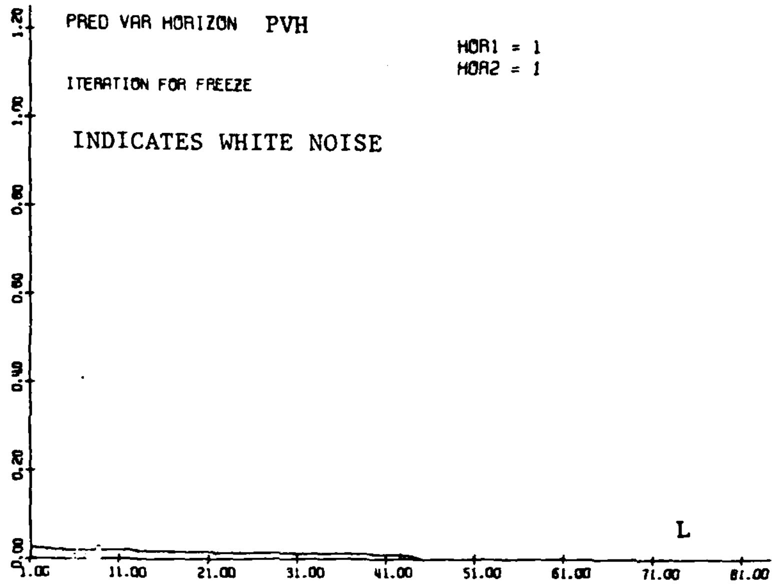


Fig. 16

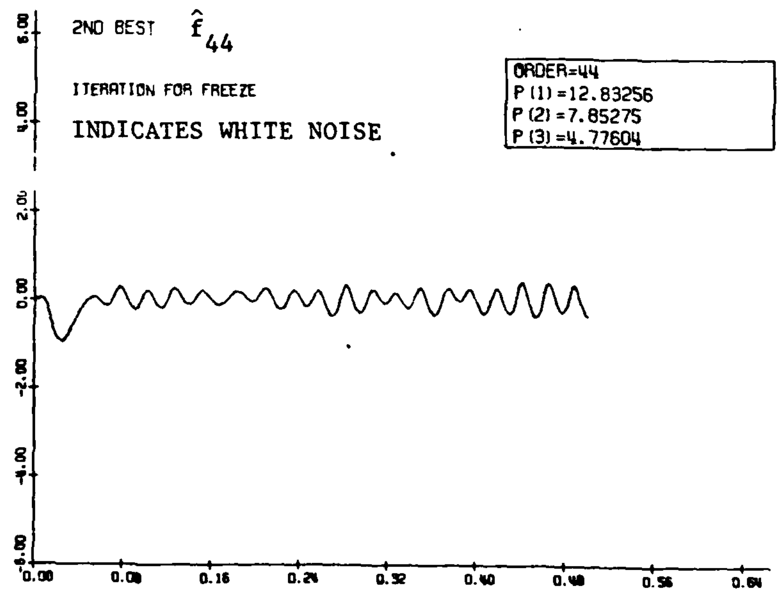


Fig. 17

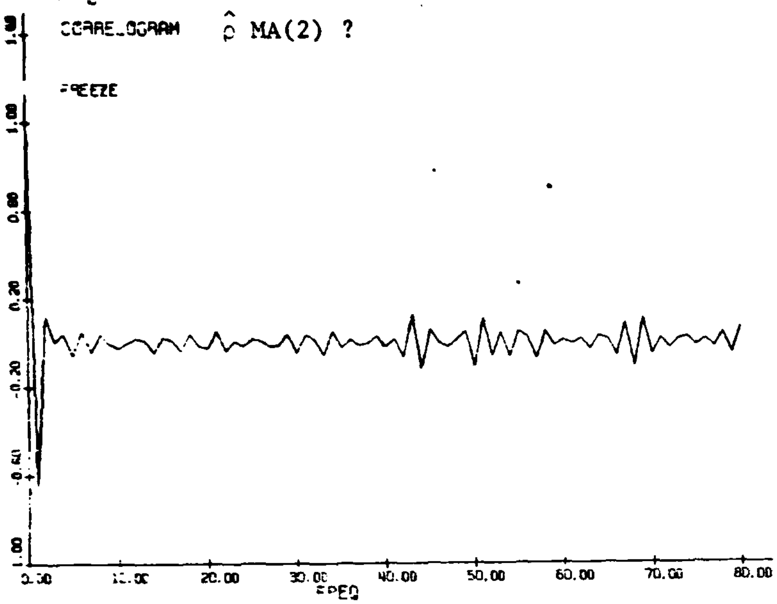


Fig. 18

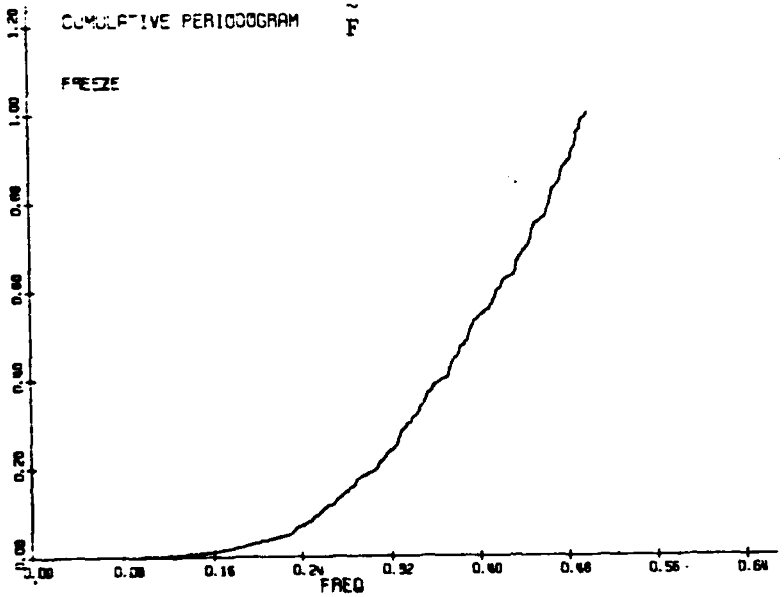


Fig. 19

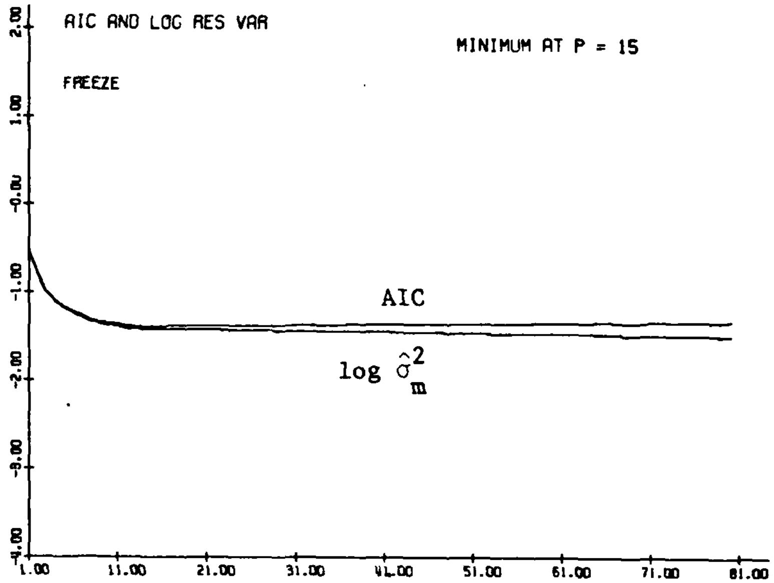


Fig. 20

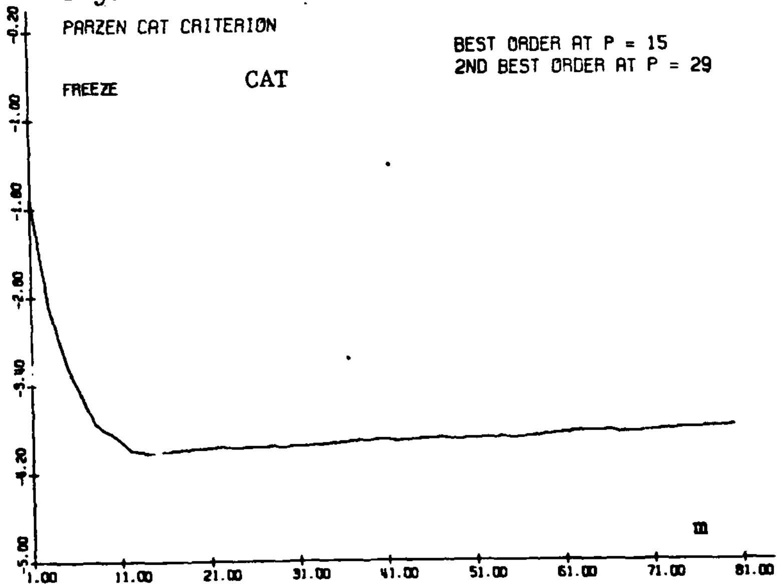


Fig. 21

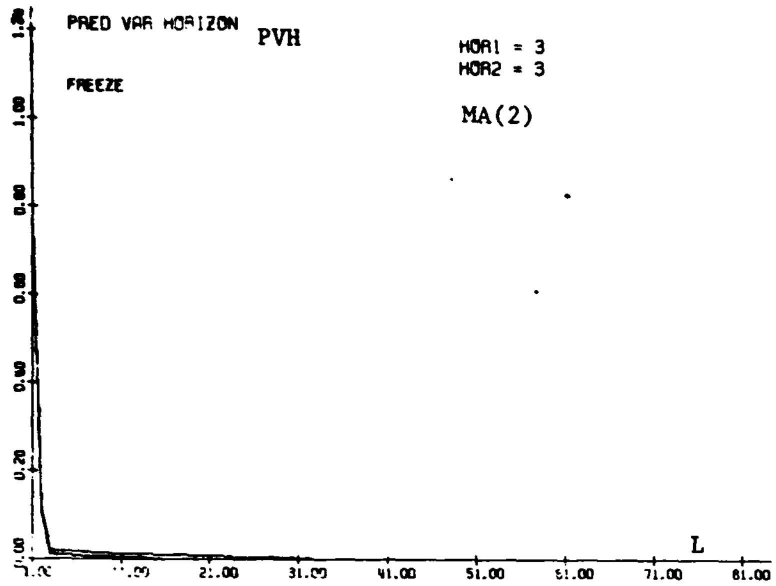


Fig. 23

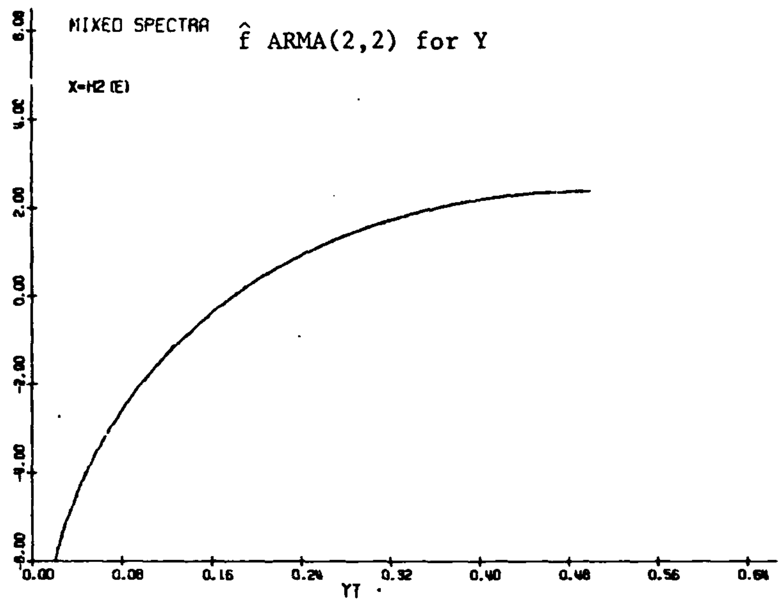


Fig. 22

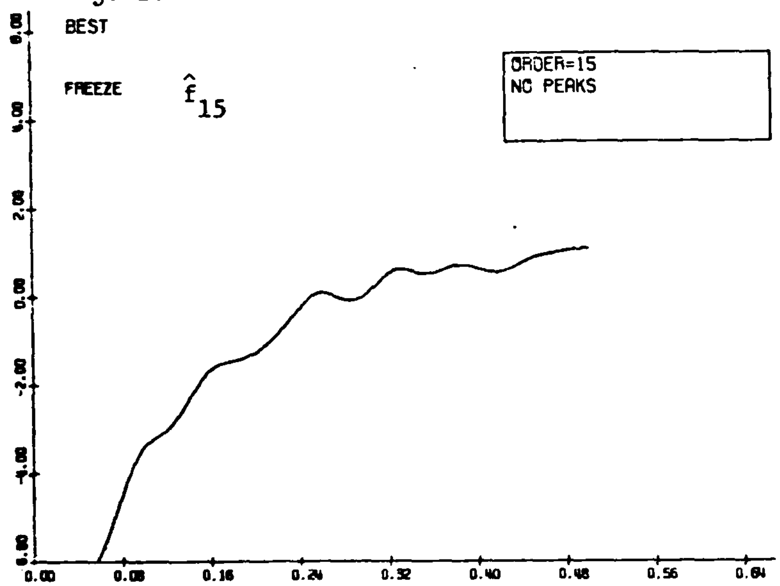


Fig. 24

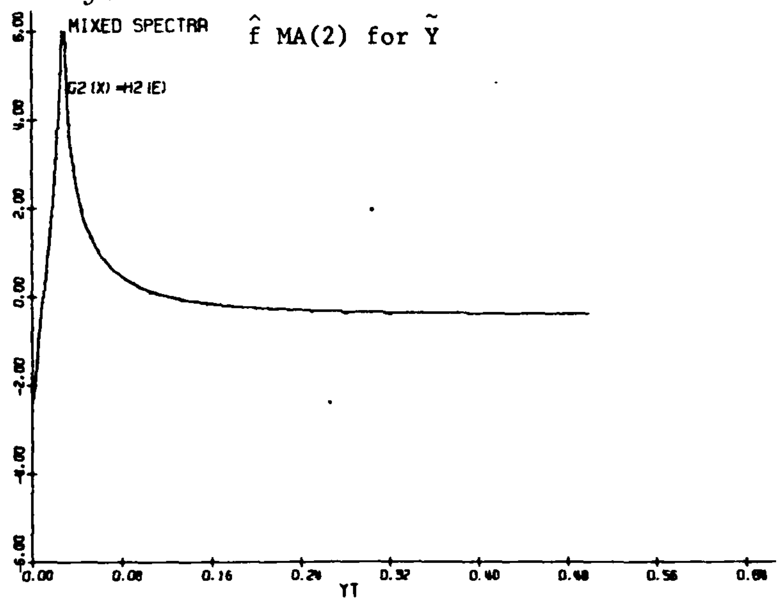


Fig. 25

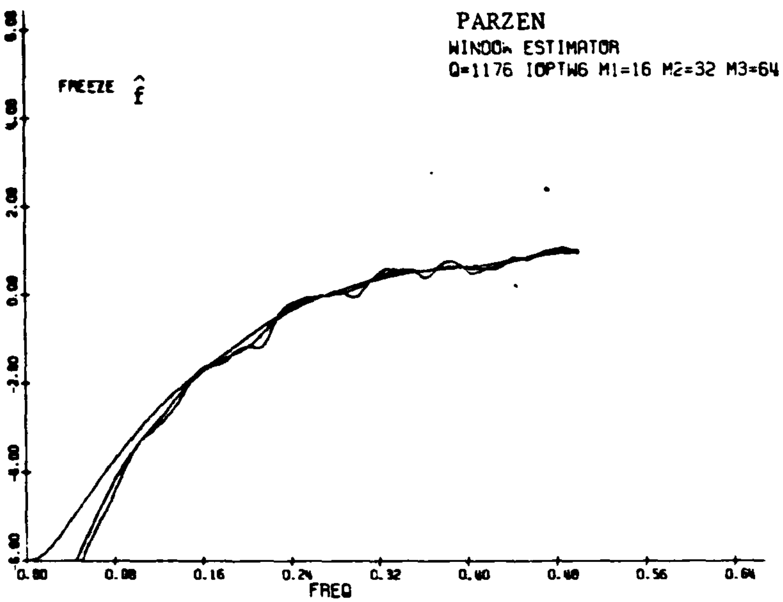


Fig. 27

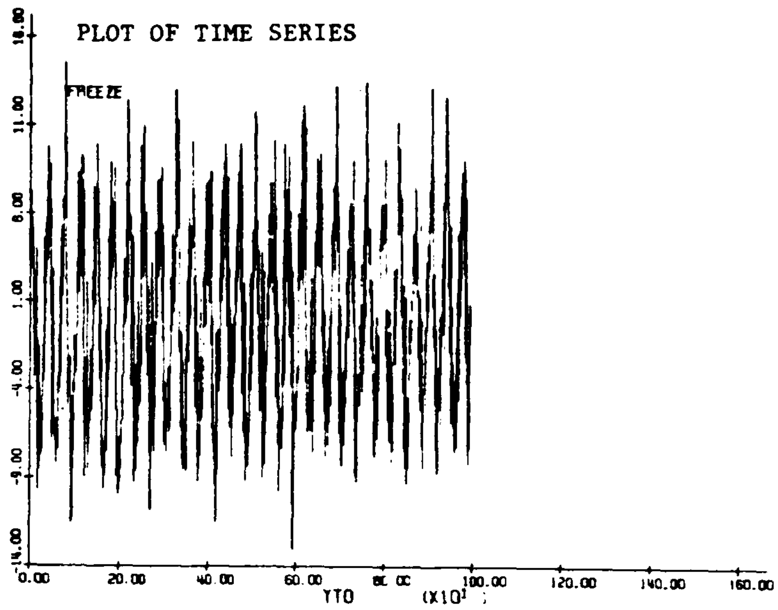
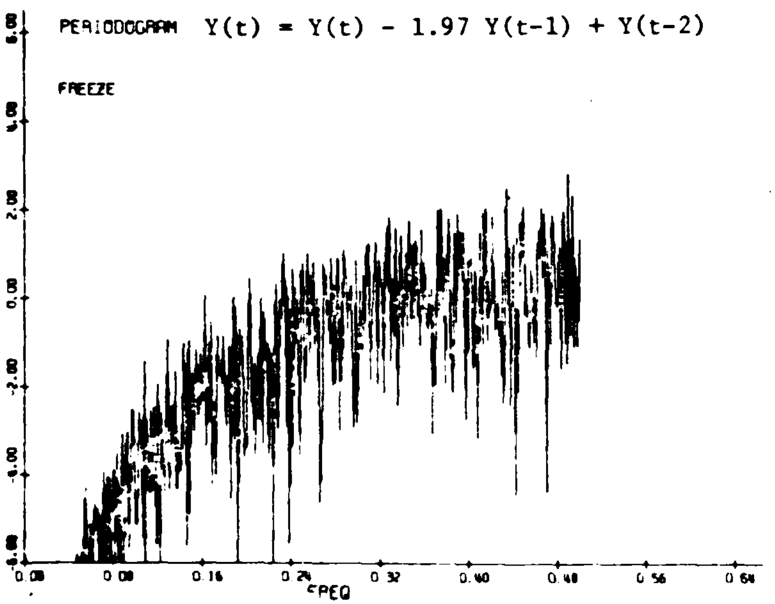


Fig. 26



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